Ünlü, A., Brandt, S., Albert, D. (2009). *Some corrections and improvements to the paper "Surmise relations between tests- mathematical considerations".* Unpublished manuscript.

Some corrections and improvements to the paper "Surmise relations between tests—mathematical considerations"

Ali Ünlü *

Department of Mathematics, University of Augsburg, D-86159 Augsburg, Germany

Silke Brandt, Dietrich Albert

Department of Psychology, University of Graz, A-8010 Graz, Austria

---

This corrigendum formulates some corrections and improvements to the paper "Surmise relations between tests—mathematical considerations" [*Discrete Applied Mathematics* 127 (2003) 221–239].

A misconception is involved by introducing the terms 'surmise relation between tests,' 'left-covering surmise relation between tests,' and 'right-covering surmise relation between tests.' This terminology misleadingly suggests that left- and right-covering surmise relations between tests are necessarily surmise relations between tests, in the sense that they satisfy the axioms defining a surmise relation between tests. Unfortunately, this is not true in general. What is solely true, as also described in the paper (Lemmata 9 and 13), is that the left- and right-covering surmise relations between tests are subsets of the surmise

* Corresponding author. Tel.: +49 821 598-2236; fax: +49 821 598-2200.
Email address: `ali.uenlue@math.uni-augsburg.de` (Ali Ünlü).

relation between tests, but neither satisfies the definition of a surmise relation between tests (Definition 5). This misconception even led to the inadequate 'proofs' of Corollaries 10 (p. 226) and 14 (p. 228). The adequate proofs have to be directly based on the definitions of the left- and right-covering surmise relations between tests (Definitions 8 and 12, respectively). Reflexivity of the left-covering surmise relation between tests: For any $A \in \mathcal{T}$ and $a \in A$, it holds $a \in A \cap \bigcap \mathcal{K}_a =: A_a$, and $A_a \neq \emptyset$. Reflexivity of the right-covering surmise relation between tests: For any $A \in \mathcal{T}$, it holds $\bigcup_{a \in A} A_a = A$.

The whole discussion in Section 3 can be extended to the general case of an *arbitrary* domain Q, knowledge structure $\mathcal{K}$, and test set $\mathcal{T}$. 'Arbitrary' here means that these sets may be of any cardinality, including uncountably infinite. An improved and generalized formulation is as follows: The *test knowledge state* corresponding to a knowledge state $K \in \mathcal{K}$ is defined as the $|\mathcal{T}|$-tuple $\dot{K} := (T \cap K)_{T \in \mathcal{T}}$. The set $\dot{\mathcal{K}} := \{\dot{K} := (T \cap K)_{T \in \mathcal{T}} : K \in \mathcal{K}\}$ is called the *test knowledge structure*. For an *arbitrary* family $\mathcal{F} := \{\dot{K} := (T \cap K)_{T \in \mathcal{T}} : K \in \mathcal{I}\}$ $(\mathcal{I} \subset \mathcal{K})$ in $\dot{\mathcal{K}}$, the *union* and *intersection* of $\mathcal{F}$ are respectively defined by

$$\dot{\bigcup}\mathcal{F} := \left(T \bigcap \cup_{K \in \mathcal{I}} K\right)_{T \in \mathcal{T}},$$
$$\dot{\bigcap}\mathcal{F} := \left(T \bigcap \cap_{K \in \mathcal{I}} K\right)_{T \in \mathcal{T}}.$$

At this point, it is important to note that the concepts of union and intersection of test knowledge states play an essential role in this paper (e.g., Definitions 17 and 21).

Using this improved and generalized formulation, for instance the proof of Lemma 19 (p. 229), which again constitutes a central result of this paper, can be considerably simplified. The proof of this important result then reduces to:

For any family $\mathcal{F} := \{\dot{K} := (T \cap K)_{T \in \mathcal{T}} : K \in \mathcal{I}\}$ $(\mathcal{I} \subset \mathcal{K})$,

$$\dot{\bigcup}\mathcal{F} \in \dot{\mathcal{K}} \Longleftrightarrow \cup_{K \in \mathcal{I}} K \in \mathcal{K},$$
$$\dot{\bigcap}\mathcal{F} \in \dot{\mathcal{K}} \Longleftrightarrow \cap_{K \in \mathcal{I}} K \in \mathcal{K}.$$

Conceptually, this is much more accessible than the number of rather technical lines of the proof presented in the paper.

Some minor corrections: In the caption to Figure 7 (p. 227), replace '$A\ \dot{\mathcal{S}}_r\ B$ and $C\ \dot{\mathcal{S}}_r\ D$' by '$B\ \dot{\mathcal{S}}_r\ A$ and $D\ \dot{\mathcal{S}}_r\ C$.' In the 'proofs' of Corollaries 10 and 14 (see above corrections), Proposition 6 was erroneously referred to as Lemma 6. On page 230, Lemma 19 was erroneously referred to as Corollary 19.